Level set based Robust Topology Optimization of Electromagnetic System under Loading Uncertainty

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This paper proposes level set (LS) based robust topology optimization (RTO) method that can be used for electromagnetic (EM) design problem under uncertain loads (UL). For this purpose, a weighted summation of expected value and variance of EM energy is used for an objective function. Since traditional stochastic methods to solve this problem with continuous probabilistic density function (PDF) are computationally intractable, thus analytic approach is proposed for calculation of stochastic performance measure for normally distributed conditions. The problem for UL is equivalent to multiple load case and can be solved efficiently and accurately by a small set of auxiliary problems. The suggested method is implemented to c-core actuator problem considering uncertainty of coercive force (CF).

***Index Terms*— Coercive force, Level set, Robustness, Topology optimization, Uncertainty**

I. Introduction

Topology optimization allows one to obtain a conceptual design using a technique for optimal material distribution in a given design domain without depending on a priori knowledge. The importance of robust optimum design is widely recognized in the field of EM design considering the effects for uncertainties of design variables.

The LS based TO has been applied to the optimal ferromagnetic distribution of c-core actuator system [1]-[2]. The minimum compliance problem having multiple loads is applied to calculate sensitivity and performance for UL problem [3]-[4]. The reliability based topology optimization (RBTO) for UL is introduced by Wang et al [5].

In this paper, we present a LS based RTO (LSRTO) which consider analytic approach of combined expected value and variance of magnetic energy for uncertain CF.

II. LSRTO under Loading Uncertainty

A. LSRTO Problem

The LS method is a numerical technique using an implicit function for tracking interfaces and boundaries. The principle of LS based TO is to update the implicit function using a velocity function derived from the shape sensitivity, so that the design progresses iteratively toward an optimum.

The structure is defined by an implicit function $\Phi(x)$ so that its zero LS coincides with the boundary,

$$\begin{cases} \Phi(x) > 0, & \forall x \in \Omega_D \\ \Phi(x) = 0, & \forall x \in \Gamma_D \\ \Phi(x) < 0, & \forall x \notin \Omega_D \end{cases} \tag{1}$$

where Ω_D is the design domain, x stands for an arbitrary position in design one and Γ_D is the boundary by zero LS.

The Hamilton–Jacobi equation for updating the LS function that represents the material boundary changes is defined as

$$\frac{\partial \Phi}{\partial t} + V \cdot \nabla \Phi = \frac{\partial \Phi}{\partial t} + V_n |\nabla \Phi| = 0 \tag{2}$$

where V and V_n are velocity vector and velocity function acting normal to interface.

The purpose of the LSRTO problem is to find an optimal shape of design domain to maximize the robust objective function, Ψ_{RB} consisting of EM energy, Ψ_{EM} under UL subject to a volume constraint $\mathrm{Vol}_{\max}$. The LSRTO is defined in a following form

$$\underset{\Gamma_D}{\text{Maximize}}\ \Psi_{RB} = \alpha E[\Psi_{EM}] - (1-\alpha)\sqrt{Var[\Psi_{EM}]} \tag{3}$$

$$\text{s.t. } a(A, \bar{A}, \Phi) = l(\bar{A}, \Phi),\ \forall \bar{A} \in \tilde{A} \tag{4}$$

$$V(\Phi) = \int_{\Omega_D} H(\Phi)\, d\Omega \le \mathrm{Vol}_{\max} \tag{5}$$

where $\alpha \in [0,1]$ is a weighting factor for the two parts of the objective, $a(\bullet)$ and $l(\bullet)$ are the energy bilinear form and load linear one of magnetostatic field respectively, A and $\bar{A}$ are magnetic vector potential and virtual one, $\tilde{A}$ is the space of admissible virtual vector potential, V is the volume, $H(\bullet)$ is the Heaviside function and Ω is the design domain larger than Ω_D which is a subset of Ω.

B. Robust Objective using Expected and Variance of Magnetic Energy under UL

The robust objective (3) is constructed as a weighted sum of expectation and variance of objective, which, in this case, is the EM energy. The square root of variance is applied to eliminate disparity of units between expected value and variance.

The expected objective function $E[\Psi_{EM}]$ under UL is derived as follows

$$E[\Psi_{EM}(J)] = \int_{J_n} \cdots \int_{J_1} \Psi_{EM}(J) \prod_{i=1}^{n} P(J_i) dJ_1 \cdots dJ_n \tag{6}$$

where J stands for applied load by the CF, $P(J_i)$ is the PDF for i th UL and n is the number of UL.

A discretized expression for EM energy can be written as

$$\Psi_{EM}(J) = \frac{1}{2}\sum_{\Omega_O}\{J\}_e^T[K]_e^{-1}\{J\}_e \qquad (7)$$

where $\{J\}$ is the CF loading vector for discretized form of (4), $[K]$ is a symmetric magnetic stiffness matrix with order which is the number of degrees of freedom, subscript e stands for element discretized form and Ω_O is the objective domain.

Assuming that the UL are uncorrelated and Gaussian distribution, the expected value of EM energy (6) can be derived analytically using integration by parts [4].

$$E[\Psi_{EM}] = \frac{1}{2}\sum_{\Omega_O}\left[\{J_\mu\}_e^T[K]_e^{-1}\{J_\mu\}_e + \sum_i^n\{J_{\sigma,i}\}_e^T[K]_e^{-1}\{J_{\sigma,i}\}_e\right] \qquad (8)$$

where J_μ stands for the mean magnitude of UL, $J_{\sigma,i}$ is the standard deviation of one. The expected EM energy (8) is equivalent to the problem for $1+n$ multiple deterministic loads can be solved analytically by a small set of auxiliary problems. The first load case is the simultaneous application of mean CF J_μ and the subsequent n cases correspond to a single one equal to $J_{\sigma,i}$ applied at the location of a i th UL [3]-[4].

The variance of Ψ_{EM} can be derived by evaluating the following

$$Var[\Psi_{EM}(J)] = \int_{J_n}\cdots\int_{J_1}\Psi_{EM}(J)^2\prod_{i=1}^{n}P(J_i)dJ_1\cdots dJ_n - E[\Psi_{EM}]^2. \qquad (9)$$

For a normal distribution of UL, (9) can be derived analytically using integration of normal PDF by parts and squaring expected value of EM energy (8).

$$Var[\Psi_{EM}] = 2\{A_\mu\}^T[\bar{\sigma}]\{A_\mu\} + \sum_{i=1}^{n}\{A_i\}^T[\bar{\sigma}]\{A_i\} \qquad (10)$$

where A_μ and A_i stand for vector potentials for CF J_μ and $J_{\sigma,i}$ respectively and $[\bar{\sigma}]$ is a diagonal matrix with standard deviation of CF.

C. Shape Sensitivity for Normal Velocity of LS Method

Using the design sensitivity analysis, the speed function V_n which defines the propagation of all LS of the embedding function $\Phi(x)$ along the normal direction of the implicit moving boundary can be calculated as

$$\frac{\partial\Psi_{RB}}{\partial\Phi} = \int_{\Gamma_D}\left[\alpha\frac{\partial E[\Psi_{EM}]}{\partial\Phi} - (1-\alpha)\sqrt{\frac{\partial Var[\Psi_{EM}]}{\partial\Phi}}\right]V_n d\Gamma_D \qquad (11)$$

$$\frac{\partial E[\Psi_{EM}]}{\partial\Phi} = \{A_\mu\}^T\frac{\partial[K]}{\partial\Phi}\{A_\mu\} + \sum_{i=1}^{n}\{A_i\}^T\frac{\partial[K]}{\partial\Phi}\{A_i\} \qquad (12)$$

$$\frac{\partial Var[\Psi_{EM}]}{\partial\Phi} = \{A_\mu\}^T\frac{\partial[K]}{\partial\Phi}\{\lambda_\mu\} + \sum_{i=1}^{n}\{A_i\}^T\frac{\partial[K]}{\partial\Phi}\{\lambda_i\} \qquad (13)$$

where (12) and (13) are the shape sensitivity for expected and variance of objective function and are calculated using adjoint vector potential λ_μ and λ_i by adjoint variable method [1]-[2]-[4]. The analytical formulations derived in (8), (10), (12) and (13) require $1+n$ load cases to compute the objective functions and the required sensitivities. Therefore, the computational cost of the objective and sensitivities scale linearly with the number of loads with uncertainty [4].

III. Numerical Investigation for C-Core Magnetic Actuator System

To validate optimization approach, a c-core actuator is suggested [1]-[2]. The optimization problem defined in (3)-(5) is applied to obtain the optimal topology of the yoke of the conventional c-core actuator system. The initial design of the actuator and the design domain where EM configuration will be generated is shown in Fig. 1. The amount of linear ferromagnetic material distribution is limited to 70% of the design domain. The air gap is specified as an objective domain where the magnetic energy is calculated. Since the energy variation is equal to the force, the magnetic energy can be defined to maximize the attraction force for the magnet to move the armature [1].

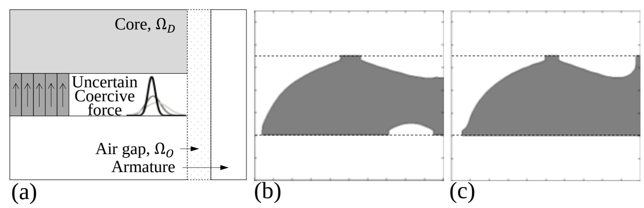

Fig. 1. C-core levitation configuration; (a) Initial design & material properties, (b) deterministic optimal design and (c) robust one (α = 0.75).

Robust results for UL have smaller variance values compared to the deterministic topology optimization (DTO) solution and there appear to be a trade-off between expected one and variance of one and are shown in Table I.

TABLE I
Comparison among Deterministic and Robust Optimal Designs

Design Solution	Expected Magnetic Energy [J]	Variance of Magnetic Energy [J]	Volume [%]
DTO	5.315×10^{-6}	1.924×10^{-4}	70.0
LSRTO (α = 0.25)	4.998×10^{-6}	1.631×10^{-4}	70.1
LSRTO (α = 0.50)	5.227×10^{-6}	1.715×10^{-4}	70.0
LSRTO (α = 0.75)	5.743×10^{-6}	1.845×10^{-4}	70.0

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